PDIG 2019-2020 REPORT: STUDENT CENTERED CURRICULUM DEVELOPMENT

1. Project Description

Describe/show to what the degree the project was carried out as planned.

This project included teachers who have already begun to shift their practice by making decisions about their teaching based on their student's learning, in addition to provincial expectations and curricular resources (e.g., workbooks). The project involved 8 teachers from 4 SWL elementary schools.

Cycle	Number of Teachers
Kindergarten	0
1	3
2	2
3	3

Description of Activities

This project initially involved six full-day working sessions with the goal of situating our mathematics curriculum within a Big Ideas framework to discern, at the Cycle level, what is essential for each topic. In addition, we looked at tasks that could be used to address these cycle-specific learning essentials. At our first meeting, teachers worked in small groups (based on teaching cycle) to establish a long-term teaching plan that sequences topics in a way that aims to build a coherent and connected understanding of mathematics. Subsequent meetings were organized based on teaching cycle. That is, teachers teaching Grade 1, 2 and 3 participated in three Lower Elementary working sessions and teachers teaching Grades 4, 5 and 6 participated in two Upper Elementary working sessions. Before the end of the year, the Lower Elementary group was supposed to meet once more and the Upper Elementary group was supposed to have two more meetings before the final meeting (whole group). The goal of the final meeting was to share and refine the math learning essentials and tasks, and revisit the long-term plan developed in the first meeting.

The table on pages 2-7 describe the activities of each group's meeting, highlighting the topics addressed, the learning essentials and tasks discussed.

Overall, this project aimed to engage teachers in ongoing professional development that incorporates collaborative learning, rethinking activities that define teaching and learning, and creating/sharing resources to share with a wider community of teachers. Added to this, opportunities to bridge research and practice supported teacher reflection about the subject matter.

LOWER ELEMENTARY GROUP		
	Topic and Learning Essentials	**Tasks[1]**

Session 1	**MEASUREMENT (CYCLE 1)**
	Length
	• Length can be measured using one small or big unit (nonstandard and standard).
	• Bigger standard units can be expressed as smaller standard units, and vice versa, because standard units of length share a multiplicative relationship.
	• Measures of length determine a number of units used to cover a distance.
	• Length can be composed using repetitions of equal-sized shorter lengths.
	  ○ Avoid gaps when iterating the unit of measurement.
	  ○ Add up parts of a length to measure the whole length.
	• Need to use identical units.
	• The inverse relationship between unit size and number of units used.
	• Measures of length are one-dimensional to measure a one-dimensional attribute of an object.
	• Estimation strategies such as chunking, or mentally using a unit similar in size but more accessible (such as the width of a finger, length of a foot etc…).
	• Ruler use: Understands that ruler are measurement tools made using equal-sized units. Using the tool involves counting units on a ruler vs. the numbers.
	
	Elapsed Time
	• Time is measured using standard big and small units that share a multiplicative relationship (seconds, minutes, hours etc…).
	• Bigger units can be expressed using smaller units, and vice versa.
	• Build understanding of time using a number line and clock.
	• Use a number line to develop a skip counting strategy (chunks of 5, 10, 15 and 30 minutes).
	• Problem solving involves word problems and calculation problems.
		MEASUREMENT TASKS
		• Let's explore the school
		• How long is your foot?
		• Sally the snail (connects concept of time and length).
		
		ELAPSED TIME TASKS
		• Build a clock from a number line
		• Solve elapsed time word problems (end time unknown; Parker & Baldridge, 2008) using https://www.mathlearningcenter.org/resources/apps
		
		MULTIPLICATION TASKS
		Activities to build fact fluency
		• Tic tac toe products (Youcubed)
		• Match Cards (Youcubed)
		• How Close to cover to 100 (also used for area and building arrays; use with coloured cubes and grid; Youcubed)
		• Backwards Bingo (TPT)
		• Dominoe war (Box cars and one-eyed jacks)
		
		Building conceptual understanding (problem types Carpenter et al. 2014; Cognitively Guided Instruction)
		• Pizza/Pepperoni (grouping; Youcubed)
		• CGI Word problems and numberless word problems
		• Dozens of Dice (grouping; Jo Boaler et al. Gr 3)
		• How close to cover 100 (array/area)
		• Visual proof (area; adapted from Jo Boaler et al. Gr 4 using 14 x 5, for example).

[1] This report includes a link to a google drive folder to access lesson plans/ppt for certain tasks.

	MULTIPLICATION (GRADE 3) Build conceptual understanding of different meanings of multiplication[2] • The **grouping** meaning represents multiplication, where the number of groups and size of each group is known, and the total amount is unknown. The measure (size) of each group is equal. • The **array** meaning represents multiplication, where the number of rows and number of columns are known and the total is unknown. • The **area** meaning represents multiplication, where the both dimensions of a rectangle are known and the area is unknown. • Strategies for multiplying whole numbers should connect counting strategies (i.e., skip counting) with operation strategies (e.g., repeated addition). • Use, connect and compare multiple representations (physical, visual, symbolic and contextual). • Developing fact fluency involves supporting the learning of math facts by using fact strategies, the relationship between multiplication and division (reverse operations), and multiplication properties (identity and commutativity). Math fact use should be highlighted regularly with the goal to help students see the relationships between fact families. Students can use math facts derived from the 2, 5, 10 and 11 fact families to solve problems involving the 3, 4, 6, 7, 8, 9 and 12 fact families. For example, 3 x 9 can be found by 3 x 10, and then removing the extra group of 3.	
Session 2	PLACE VALUE (CYCLE 1) • Numerals represent amounts using 10s and 1s. • Counting by 10s and 1s (vs 1s only). • The rules for using digits to write how many. • The role of zero. • Identifying different-sized groupings represented in a numeral. • Flexibility representing an amount using 10s and 1s. • Regrouping takes place when one larger-sized group is broken into (decomposed) 10 smaller-sized groups, and when 10 smaller-sized groups are combined (composed) to form one larger-sized group. • Use, connect and compare multiple representations (physical, visual, symbolic and contextual).	PLACE VALUE TASKS • Counting collections (see Stephanie Latimer video on YouTube) • Candy Shop address counting collections by 10s and 1s and helps students see different-sized groupings represented in a numeral (taken from McClain et al., 1998; Math Summer Institute) • Connecting different ways to represent amounts (**physical** such as proportional models, Base 10 Blocks; nonproprtional models such as the Chip model, Parker and Baldridge, 2004; **visual** models such as arrays; **symbolic; contextual,** or word problems) and comparing representations.

[2] Note that other meanings of multiplication were discussed (rate/price, comparison, and combination) however, our tasks focus on these three meanings.

	MEASUREMENT CONT. (CYCLE 1 AND GRADE 3) Money • Money is measured using standard small and big units. • Converting big units to smaller units, and vice versa. Using different denominations to make one dollar. • Strategies for quantifying a collection of money should connect counting strategies with operation strategies, and should involve concrete tools (e.g., hairy money, Base 10 blocks, and hundreds chart). • Problem solving involves word problems and calculation problems.	• Multiplication problems and Measurement division problems where the size of the group is 10 support counting collections by 10s and 1s and helps students see different-sized groupings represented in a numeral (CGI, 2014). • Open tasks as opposed to closed tasks help promote flexibility with renaming (e.g., 312 can be represented with 3 flats, 1 long, 2 units or 31 longs and 2 unit etc..). MEASUREMENT TASKS • Learning to use the 100s chart to count collections • Snack Machine 3 ACT Math • Hairy money (TPT) • Word problems total amount is unknown or difference is unknown.

Session 3	ADDITION/SUBTRACTION (CYCLE 1 AND GRADE 3)
	Build conceptual understanding of different meanings of addition and subtraction
	• The **joining/separating** meaning represents a **starting** amount that is **changed** by joining a separate amount or by separating the start amount to take a part of it away. Both actions **result** in a new amount. Problem types differ based on what is unknown.
	• The **part-whole** meaning represents a whole made up of at least two parts, not necessarily equal in size. These problems reflect a relationship between parts and a whole (vs. an action). Problem types differ based on what is unknown.
	• The **comparison** meaning two quantities being compared, a **bigger** quantity and a **smaller** quantity. There is an additive relationship between the bigger quantity and the smaller quantity (i.e., the **difference**).
	• Strategies for adding and subtracting whole numbers should connect counting strategies (e.g., counting on) with operation strategies.
	• Invented and nonstandard addition strategies should make connections to math concepts and processes, namely decomposing numbers and combining like units, place value, compatible numbers, and derived facts/compensation.
	• Use, connect and compare multiple representations (physical, visual, symbolic and contextual).
		ADDITION/SUBTRACTION TASKS
		Activities to build fact fluency
		• Tic tac toe sums (Youcubed)
		• Snap It (Youcubed)
		• Pig (Math for Love)
		• Totality for Two (Nrich)
		
		Building conceptual understanding (problem types Carpenter et al. 2014; Cognitively Guided Instruction)
		• CGI Word problems and numberless word problems
		• 3 ACT Math
		  ○ https://gfletchy.com/humpty-dumpty/
		  ○ https://gfletchy.com/bag-o-chips/
		  ○ https://gfletchy.com/the-cookie-monster

	• Developing fact fluency involves supporting the learning of math facts by using fact strategies, the relationship between addition and subtraction (reverse operations), and addition properties (identity and commutativity). Math fact use should be highlighted regularly with the goal to help students see the relationships between fact families.	
UPPER ELEMENTARY GROUP		
	Topic and Learning Essentials	Tasks[3]
Session 1	PLACE VALUE (GRADE 4 AND CYCLE 3) • Decimal numbers are another way of writing fractions. Understanding the value of digits positioned to the right of the decimal point is based on understanding how decimal numbers relate to fractions. • Use, connect and compare multiple representations (physical, visual, symbolic and contextual). • Flexibility renaming quantities. • Flexibility representing an amount to the right of the decimal point using both proportional (e.g., base-ten blocks) and non-proportional (e.g., chips) models. • Understanding the role of zero. • Identifying different-sized groupings represented in a numeral, including numbers with digits in the tenths or the hundredths position. • Adjacent places are related to each other by a factor 10. This illustrates how places grow exponentially by factors of 10 and that all whole numbers are represented by groups of 10 and 1s. • Revising rules used to understand number magnitude for whole numbers to decimal numbers. THE MEANING OF FRACTIONS (GRADE 4 AND CYCLE 3) • A fraction is any number that can be written as a/b. Both a and b can be any number included in the set of integers, with the exception that b cannot be 0. • For all three meanings of a fraction, the numerator and denominator share a multiplicative relationship. • Identify the fraction based on the whole, and whole using the fraction. • Part-whole: A fraction expresses a part in relation to its whole unit. The whole	PLACE VALUE TASKS • Measurement division problem (include number pairs; number of groups is a whole number and the size of each group is 1/10; Empson & Levi, 2011) • Cover it up! (change numbers) • Represent decimal numbers using a variety of visuals and manipulatives to illustrate the place value of digits in positions to the right of the decimal point and their relationship to the ones (e.g., 100-grid, Base 10 blocks) • Decimals on a Line (Boaler et al. Gr 4, 2017) • Can you make it? (Boaler et al. Gr 4, 2017) • Decimal card sort • Dice Game FRACTIONS TASKS (BASIC CONCEPTS AND MEANINGS) Division • Using Equal Sharing problems (see chapter from Empson and Levi, 2011 re: problems, instructional guidelines by level, and student strategies) to develop the division meaning of fractions and help build understanding that the numerator and denominator are related, not separate. • Three ACT Math lesson: Cheese and crackers

[3] This report includes a link to a google drive folder to access lesson plans for certain tasks.

	(denominator) can represent a continuous quantity (shown visually with an area or length model) or a group of discrete (countable) things (set model). The denominator indicates the number of equal-sized parts that together make up each whole unit. The denominator indicates the size/value of the numerator. • **Division (Cycle 3 only)**: A fraction represents the quotient of two numbers. This meaning shows students that it is possible to divide a smaller number by a bigger number. This meaning also explains why you cannot divide a number by 0. • **Ratio (Cycle 3 only)**: A fraction compares a number of units used to represent one quantity relative to another quantity. The comparison can be part-to-part or part- to-whole. COMPARING AND ORDERING FRACTIONS (GRADE 4 AND CYCLE 3) • Equivalent fractions is central to building understanding of basic fraction concepts. • Fractions can be represented in infinitely different ways, without its value changing. Equivalent fractions share the same mathematical relationship and describe the same amount/quantity using different-sized fractional parts. Renaming fractions (representing a mixed number as an improper fraction or finding an equivalent fraction) is sometimes necessary when operating on fractions. • Fractions can be compared by reasoning about the relative size of the fractional parts. The size of the parts depends on the number of parts within the whole. As the number of fractional parts increases within the whole, the size of each part decreases. • Fractions can also be compared using a benchmark (½ or 1).	Part Whole • Activities from Boaler et al. Gr 3 (2018) to identify the fraction of a whole, and using the fraction to identify-build its whole: o I spy ½ o The many shapes of ½ o Seeing parts and wholes o Cover, cut, and sort • Activities from Boaler et al. Gr 4 (2017) to identify the fraction of a whole: o Pixeled fractions • Open questions (using any of the materials in the bag, represent a whole and *its* half). • Question using length model to use the fraction to identify-build its whole. • Find all possible rectangles with a fixed perimeter where the length of at least two sides represents a fraction (e.g., 2 1/2). Ratio • Word problems taken from Parker and Baldridge • Nanna's paint mixup COMPARING AND ORDERING FRACTIONS TASKS • Using Equal Sharing problems (Empson and Levi) where the size of each group can result in different, but equivalent fractions, depending on the sharing strategy used. • Equivalencing problems (Empson and Levi) which involves two equal sharing situations. The number of shared items for one of the equal sharing situations is unknown. Both equal sharing situations are related because the mathematical relationship between the number of sharers and number of items shared is equal. • Comparison problems (Empson and Levi) which involves two equal sharing situations. The number of

		items shared and number of sharers is known for both situations. The student must determine mathematical relationship between the number of sharers and number of items shared in both situations is equal. • Dominoes war • Fraction card sort
Session 2	IMPLEMENTING A PROBLEM-SOLVING REASONING TASK (CYCLE 2) OPERATIONS WITH FRACTIONS (CYCLE 3) • The meaning of each of the operations is the same with whole numbers as fractions. • Addition: The first addend represents the starting amount. The second addend represents the amount added to the starting amount. Addends can also represent two parts of a whole unit. The sum is the result of the combination of fractional parts. • Subtraction: The subtrahend represents the starting amount. The minuend represents the amount taken away from the starting amount. The minuend and difference can also represent two parts of a whole unit. • Multiplication: One factor represents the number of groups and the second factor represents the size of each group. When the number of groups is a whole number, the product represents the combination of all fractional parts. • When the fractional parts have different values, identifying equivalent fractions allows for the combination of fractional parts. • When adding and subtracting fractions, the standard algorithm involves adding and subtracting fractions with a common denominator based on the idea of combining/subtracting fraction parts that are all the same size (fractional parts that relate to the same whole unit). However, fractions with different but related parts (for example, halves, fourths, and eighths; or, thirds, sixths, and ninths etc...) can be combined/subtracted without finding a common denominator. The sum/difference would be represented using the smallest common unit fraction.	OPERATIONS WITH FRACTIONS TASKS • Plinko! (Get your Teach on): Students match word problems to the correct fractions addition/subtraction number sentence. Students play the game a second time but are required to write a word problem to match a fractions addition/subtraction number sentence.

Include what went well and what proved to be a challenge.
The project was unable to carry out all six working sessions, which limited the number of topics addressed as well as time to reflect and refine what was addressed. Although I did not have the opportunity to collect feedback from all teachers, I did receive positive feedback from some of the teachers and 4 of the 7 teachers would like to continue this project next year.

Teachers appreciated the collaborative nature of the project and valued learning about what other teachers do in their classroom. Breaking up the group into a Lower Elementary and Upper Elementary group allowed for deeper discussions of the math content and promoted teacher participation.

Finding an effective structure/routine for the sessions to support the project's goals took time to develop. This issue along with the missed opportunity to continue and complete all six sessions, I believe, hindered opportunities to focus on some of NCTM's (2014) eight effective teaching practices, namely

- Shift 1: Establish mathematics goals to focus learning.
- Shift 2: Implement tasks that promote reasoning and problem solving.
- Shift 3: Use and connect mathematical representations.
- Shift 4: Facilitate meaningful mathematical discourse.
- Shift 5: Pose purposeful questions.
- Shift 6: Build procedural fluency from conceptual understanding.
- Shift 7: Support productive struggle in learning mathematics.

Moving forward with this project next year, I plan to incorporate the practice of returning to these shifts at every working session to enhance discussion of math learning essentials and the analysis of tasks.

Include a synthesis of your journal entries
The journal entries included a summary of the topics addressed at each working session as well a brief description of the tasks analyzed. Added to this, any resources shared were outlined. The entries also summarize future goals/next steps for upcoming meetings.

Resources Shared

Building Classroom Culture
https://www.saravanderwerf.com/100-numbers-to-get-students-talking/ .
Classroom Routines to Support the use of Problem Solving

- https://earlymath.erikson.edu/exploring-3-reads-math-protocol-word-problems/
- Problem Solving Strategies Poster
- Talk Moves poster

Number Talk ideas

- **To reason about multiplication** https://www.youtube.com/watch?v=wxE2Kur4AHc

- **Develop Fact Fluency Strategies**
  https://bstockus.wordpress.com/2019/09/26/multiplication-number-talks-using-models/

Warm up

- https://www.cemc.uwaterloo.ca/resources/potw.php
- https://www.youtube.com/watch?v=srJWx7P6uLE
- https://www.mathreasoninginventory.com/Home/Resources (see assessment pdf files for whole numbers, decimals and fractions)

Templates and Virtual Manipulatives

- http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html
- https://www.mathlearningcenter.org/resources/apps
- http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm

Resources for Rich Tasks

- https://www.mathreasoninginventory.com/Home/Resources
- https://tapintoteenminds.com/3act-math/
- https://gfletchy.com/3-act-lessons/
- https://whenmathhappens.com/3-act-math/
- https://www.youcubed.org/
- https://nrich.maths.org/

Jo Boaler, Jen Munson and Cathy Williams. Mindset Mathematics: Visualizing and Investigating Big Ideas, Grade 3 (2018), 4 (2017), 5 (2018) and 6 (2019).

Parker, T. H., & Baldridge, S. J. (2004). *Elementary mathematics for teachers.* Okemos, MI: Sefton-Ash.

Research-related Resources

- Englard, L. (October, 2010). Raise the bar on problem solving. *Teaching Children Mathematics,* pp 156-165.)
- Empson, S. B. and Levi, L. (2011). Extending Children's Mathematics Fractions and Decimals: Innovations in Cognitively Guided Instruction.
- Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school.* Portsmouth, NH: Heinemann.

2. Project Goals and Outcomes

Describe/show to what degree the goals of the approved project were met.

In line with the objectives of the project, the goals were the following:

- developing and sharing new resources
- deepening subject matter knowledge for teaching
- learning from teaching as a part of a community

Describe or show the gains that the participating teachers achieved through this project.

Gains in a deeper understanding of math content for teaching

Although we were unable to complete the project this year, the focus on unpacking a mathematics topic and aligning this with provincial objectives supported rich discussions about the subject, refining, I believe, the teacher's specialized knowledge for teaching mathematics.

The math learning essentials outlined in the table on pages 2 to 7 underscore the extent to which the topics addressed were discussed. Further, some of the tasks referenced in the table are described in more detail in topic/lesson planning documents included in the group's Google Drive (link available on p. 12). The Lower Elementary group developed four lesson plans based on our discussions about establishing learning goals, and anticipating and responding to student thinking. The Upper Elementary group developed four planning documents and focused on connecting the tasks to our curriculum, analyzing the tasks in terms of its math focus, and task sequencing. Also, the Upper Elementary group collaborated to create a slide deck for a situational problem to highlight the process for launching complex tasks (Jackson et al., 2012) and ensuring high student participation. Specifically, the process emphasized having students build the question (e.g., Dan Meyer), allowing for productive struggle (e.g., Lynch et al., 2018, Jo Boaler and Dan Finkel), and differentiating the task to ensure equitable access to learning the mathematics (Lynch et al., 2018). Together, these documents provide additional insight into the teacher's developments in subject-matter knowledge.

Sharing and Using Research to Support the use of Tasks that Promote Problem Solving and Reasoning

Our discussions of mathematics content guided the selection and evaluation of tasks that would support the math learning essentials we outlined. These discussions provided opportunities for teachers to share teaching strategies and opportunities to integrate research-based recommendations. The working sessions supported teacher's participation in a community of learning which is essential to build a common vision for effective mathematics teaching that provides equitable access to math learning for all students. Equally valuable, however, was the opportunity to discuss, from a research perspective, effective ways to represent mathematics in the classroom. Planning lessons around research bridges research and practice, and stands to positively impact teacher's instructional decisions (van Es & Sherin, 2008).

The tables on page 11 describe the number of tasks discussed and how many of those tasks prompted a discussion of evidence-based practices.

Lower Elementary Group		
	Number of Tasks	**Number of Tasks linked to Research**
Measurement Length	**3**	**1** (Clements & Sarama, 2014;
Measurement Time	**2**	**2** (Math Summer Institute; Parker & Baldridge, 2008;)
Measurement Money	**4**	**3** (Dan Meyer; NCTM, 2014; Parker & Baldridge, 2008)
Place Value	**5**	**3** (Carpenter et al., 2014; Mclain et al., 1998 ; Parker & Baldridge, 2004)
Addition/Subtraction	**8**	**4** (Boaler et al., 2015; Carpenter et al., 2014 ; Dan Meyer; NCTM, 2014)
Multiplication	**10**	**3** (Boaler et al., 2015; Carpenter et al., 2014; NCTM, 2014)

Upper Elementary Group		
	Number of Tasks	**Number of Tasks linked to Research**
Place Value	**7**	**4** (Boaler et al., 2017, 2018; Dan Meyer; Empson & Levi, 2011; Rep XXXX)
Fraction Concepts and Meaning	**12**	**5** (Boaler et al., 2017, 2018; Dan Meyer; Empson & Levi, 2011; Parker & Baldridge, 2004; Rep XXXX)
Comparing and Ordering Fractions	**5**	**2** (Empson & Levi, 2011; NCTM, 2014)
Operations Fractions	**1**	**1** (NCTM, 2014)

4. Reinvestment

Clearly describe how the resources created and/or the learning achieved by the participants can be of benefit to the educational community at large.

Reinvestment of what was produced (i.e., the math lessons) this year will be included in online resource that will be made available for the upcoming school year, extending the resources to all teachers. The idea to use an online platform (Google Site) only emerged at the last Upper Elementary working session. As such, we were able to start developing this resource but were unable to include and refine all the tasks addressed. Although the site is not ready to share now, this link to this folder SCCD PDIG 2019-2020 includes these documents we developed:

- Lesson plans
- Lesson slide decks
- Teaching without a Workbook Site (in progress)